Review Comments on “Simultaneous Hand-Eye and Robot-World Calibration by Solving the AX=YB Problem without Correspondence” for RA-Letter

**Decision: ACCEPT**

**Reviewer 1** N/A.

**Reviewer 2**

This paper deals with the AX=YB calibration problem, which is well-known when A's and B's are matched. It is a good idea to propose to somewhat relax this constraint by solving the problem when there is a constant time-shift between the hand and the eye motion.  
  
Despite my comments below, I do believe this is a real contribution to the state of the art. However, the paper is rather hard to read. I would suggest to leave it as it is for ICRA but to use the remaining 2 pages before reaching the RA-L limit to develop all the technical aspects and answer some of my comments below.  
  
**ANS: Following the reviewer’s comments the manuscript has been extended by two pages and the additional comments below have been addressed.**

Now, for a scientific discussion. The numerical results consist only of a simple index-shift (\tau\_shift is an integer index and not a fractional time shift) in the Ai's and Bi's. Thus, the proposed  
framework is far too complex in this case because one could simply solve \forall i A\_{i+\tau\_shift} X = Y B\_i for all potential integer \tau\_shift's and choose for the (X,Y) pair which best fits the Euclidean invariant properties (23).   
  
Or even simpler, to solve the AX=XB problem, by first matching the displacements A's and B's (see minor comment below) using the Euclidean invariant properties (i.e. (23) applied to the displacements)  
and then solving conventionally the problem. These alternatives are not really discussed in the paper.  
  
Therefore, one would really like to see results in the case which motivates the study: asynchrony between the hand and the eye pose acquisition, that is not a simple shift in the i's. Otherwise, the proposed approach cannot be considered as validated.

**ANS: A new section is added to handle the data sets which is scrambled instead of shifted. A new method is presented to show that no correspondence is needed to recover to calculate X and Y. Numerical simulations are performed between the new method and the traditional Li’s method.**   
  
Also, one would really like to see experimental results, in a relevant set up.

**ANS: Considering the time frame for submitting the final manuscript, the authors think it might be better to leave the experiment for future publications along with other potential new results.**   
  
By the way, I like the idea of using the Fourier transform to find the time shift between the angles and pitches, but I would apply it on the displacements, not on the poses (ie. in the AX=XB case, rather than on the AX=YB case) because it seems to me more relevant to "time match" two "velocity profiles" rather than two "trajectories". Indeed, the velocity profiles neither depend on the rotation axis nor on the screw origin, contrary to the trajectories. Moreover, this would also allow for coping with different sampling frequencies, and perhaps (through more advanced time series analysis) variable sampling and/or missing samples (that is, actual asynchrony). Then, a "simple" linear interpolation over se(3) would allow to find the appropriate A's and B's for solving AX=XB, and then finding Y.

**ANS: We appreciate the reviewer’s idea and will try implementing this in the future. One problem that we foresee with this is that when there is measurement noise in the pose data, differentiating can have the effect of decreasing the signal-to-noise ratio. Determining the extent of these effects is best addressed in a paper focusing on experimental implementations of the method. Also, since we have added the case of completely scrambled data and our method still recovers X and Y, we view this as a big strength over rate-based methods that may fail not only due to noise amplification under differentiation, but also when there are gaps in the data.**  
  
Minor comments:

- in the abstract : "due to asynchrony \_in\_"

**ANS: The typo has been corrected.**

- above (10): "substituting (8) in ???"

**ANS: It has been corrected as “... the by using the bi-linearity of convolution, add n instances of Eq.(8), and substitute Eq.(9) into the summation, and we will have: ...”**

- result plots (Fig. 2, 5, and 6) should not use lines because there is  
no result in between the integer shifts.

**ANS: The line charts are used to help the readers better differentiate the multiple errors shown in each figure. The authors agree that the results are discrete instead of continuous, but line chart might be a better choice in terms of clarity and simplicity.**

- The A's and B's in the AX=YB and AX=XB problems do not have the same  
physical meaning, one being a pose and the other a displacement.  
Therefore, mixing them as it is written in the introduction (and Fig.1)  
is rather misleading.

**ANS: Detailed descriptions can be found in both the figure and the main body of the paper. A short notice is added in the caption of the figure.**

**Reviewer 3**

General Comment:  
  
It is an original approach for AX=YB without A\_i and B\_i correspondence, resorting to a distribution of Lie Group and computing the autocorrelation via IDFT, shrew lines as the shifting in the streams of A\_i and B\_i is computed. The only critic is that the authors use the SE(3) matrix representation instead of dual quaternion, or motor algebra and the Chen´s theorem to address the issue in terms of screw theory taking advantage of the fact that the \theta\_A\_i=\theta\_B\_i and d\_A\_i=d\_B\_i remain invariant, as a result one can simplify the equation using the Chen´s theorem to L\_x=\tilde M\_A  
L\_y M\_B. Than the author can use all his statistical approach to handle Lie groups and Lie algebras. The equation will be simpler.  
  
Suggestions and typo errors:  
Page: P, Column : CL left, CR right, Row R   
  
P2, CL, R1: ... descent methods [3]-[9] add two related references:  
  
and tell that Motor Algebra was used to solve AX=YB for robotics and registration in medical robotics.  
  
1. Bayro--Corrochano E., Daniilidis K. and Sommer G, [2000]. Motor algebra for 3D kinematics. The case of the hand—eye calibration. Journal of Mathematical Imaging and Vision, vol. 13, pag. 79-99.   
  
2. 82. Rivera-Rovelo J., Herold-Garcia S. and Bayro-Corrochano E. [2008]. Geometric Hand-Eye Calibration for an Endoscopio. International Conference on Robotics and Automation, ICRA’08, Pasadena, California, May 19-23, 2008.

**ANS: The references for the above two papers have been added, as has a reference to Chen’s paper.**  
  
Eqs. 11.a to 13, are hard to understand for general readers. One must know about Lie Groups and the notation Log V(H) is unknown.

**ANS: More background knowledge is added in the introduction part to explain the notations. In addition, we added several other new references (Murray, Li, and Sastry to explain exp, log, and vee; and another of co-author Chirikjian’s books to explain properties of convolution on SE(3), and other Lie groups).**  
  
P4, Eq. 23 Is this related to the Chen´s theorem, clarify?.

**ANS: The expression MA\*MX = MY\*MB might look similar to what is shown in Chen’s theorem, but here MA and MB denote the mean of a discrete set of reference frames {A\_i} and {B\_i}. We also added Chen’s 1991 paper on invariants to our reference list. That paper is concerned about the existence and uniqueness of solution for X given two pairs (A1,B1) and (A2,B2). In our problem, one can think of the mean condition MA\*MX = MY\*MB as replacing A1X=XB1 and the covariance condition replacing A2X =XB2. Chen’s theorem does not directly apply because it is not concerned with covariance.**

P6, LC, R13 to R23. This paragraph has to be rewritten it is not clear.

**ANS: The paragraph has been rewritten.**

The experimental section lacks of an study of the author`s approach using real data and real robot devices.

**ANS: Considering the limited time frame, and the spirit of RA\_L as a rapid-publication venue,**

**we were not able to include experiments here. It will be done in the future in more application-oriented works with our clinical partners.**

**Report:**

Two reviewers conclude on B+ and A, and notice the contribution of the paper. Some improvements are suggested in the reviews. It is recommended that the authors follow the recommendations in the final version of paper for ICRA and try to extend the paper for RA-L.

**ANS: The authors have followed the comments of the reviewers and extended the original manuscript by two pages. Typos are corrected, background knowledge is added to better explain the equations in the paper, and a completely new section is added to show a new method which deals with the case the correspondence between the data stream is not recoverable (or highly difficult to recover).**